Graduate Student Summer Conference 2019
“The Mock AMS”

July 25 – July 26

Abstracts

Kübra Benli

Squares modulo p

Abstract

Let p be an odd prime number. I. M. Vinogradov conjectured that the smallest prime that is a square modulo p should be $O(p^{\epsilon})$, for any $\epsilon > 0$. Linnik and A. I. Vinogradov proved that such a prime is at most $p^{\frac{1}{4}+\epsilon}$, for each $\epsilon > 0$. In this talk, we will discuss some results on the number of prime squares modulo p that are smaller than the bound proved by Linnik and Vinogradov. We will also talk about some general results on higher power residues.

Amelia Ernst

TBA

Abstract

Jonathan Foster

Let’s talk about mathematical connections: Shifting the discourse

Abstract

Mathematical connections are often framed as a product of mathematical understanding. As a result, mathematical connections have been primarily studied as a cognitive phenomenon in previous research. In this talk, I argue that connections are not solely a cognitive phenomenon but also a discursive one. Considering mathematical connections from a discursive perspective allows one to ask new questions about the relationship the teaching and learning of mathematical connections. To illustrate mathematical connections from a discursive perspective, I use an example of one lesson from an advanced ninth grade mathematics classroom.

David Galban

Goodstein Sequences and the Kirby-Paris Theorem

Abstract

For any natural numbers m and n, where $n \geq 2$, we construct the hereditary base-n representation of m in the following way. We first write m as a sum of powers n, and then do the same for all of the exponents in this expression, and then for all of its exponents, and so on until all exponents are at most n. For example, the hereditary base-2 representation of 162 would be $2^{2^2+2^1+2^0} + 2^{2^2+2^0} + 2^1$. We define the Goodstein sequence starting at m to be the sequence G_i, where $G_1 = m$ and G_{i+1} is the natural number obtained by replacing every i in the hereditary base-i representation of G_i with $i + 1$ and then subtracting one.

Goodstein sequences can grow extremely quickly, but surprisingly, it can be shown that any such sequence will stabilize at 0 after finitely many terms. Even more surprisingly, this result is independent from the standard axioms of arithmetic. In this talk, we will prove the convergence result using techniques from set theory and sketch how one might go about showing its independence from Peano arithmetic.

Tyler Genao

Why is $e^{\pi\sqrt{163}}$ almost an integer?

Abstract

Calculations tell us that the transcendental number $e^{\pi\sqrt{163}}$ is approximately

$$262537412640768743.99999999999993$$

which is miraculously an integer to 12 decimal places. Are there any good reasons for this? As it turns out, a special modular form can provide a neat explanation when paired with a result from complex multiplication.

Ernest Guico

Gerschgorin Disks

Abstract

Eigenvalues tell us many things we would like to know about matrices. For example, the trace of a matrix is the sum of its eigenvalues, and the determinant of a matrix is their product. Using the latter, we can easily detect whether a matrix is invertible if we know its eigenvalues: just check if any of them are zero! However, we usually find eigenvalues by finding the roots of the characteristic polynomial and does anyone really want to do that? Enter Gerschgorin disks, which give us a very easy way of getting valuable information about eigenvalues without doing much work at all. Tune in for more information during this introductory talk! #OhMyGersch

Andy Jenkins

The Decomposition Map and Modular Representation Theory

Abstract

Let G be a finite group and p a prime that divides the order of G. To study modular representations for G, we frequently use information from ordinary (characteristic zero) representations, and a common way to relate these is via the theory of splitting p-modular systems due to Brauer. We will discuss how this setup can be used to reduce representations modulo p via the decomposition map d, and we will see that d fits into a diagram known as the cde-triangle. As an application, we will prove that the Cartan matrix is symmetric.

Matt Just

Ordered Partitions and Factorizations

Abstract

Ordered partitions, also known as compositions, have been studied extensively. The structure of a composition is easy to break down since the ordinary generating function for all compositions is rational. When considering ordered factorizations the multiplicative structure requires the use of a Dirichlet generating function. We apply some techniques used to study the structure of compositions to ordered factorizations, illustrating the limitations and the need for new methods.

Jinsil Lee

Mathematical analysis for daisy chain gene drives

Abstract

Recent advances in research on gene drives have produced genetic constructs that could spread a desired gene into all populations of a species, with a single release in one place. This is effective, but also comes with risks and ethical concerns. There has been a call for research on gene drive systems that are spatially and temporally self-limiting. We can analyze the daisy drive systems, spatially self-limiting gene drive systems, for the ODE case and the PDE case.

Thomas Melistas

Pseudoholomorphic curves in celestial mechanics

Abstract

Predicting orbits in space is generally a hard task. In this talk we will focus on the three body problem, or rather an easier version which is the RPCR3BP. We will introduce the problem, state the difficulties one faces and we will explain ways to approach it using symplectic and contact topology, in particular pseudoholomorphic curves, towards understanding and analyzing such an example of a Hamiltonian system.

Clay Mersmann

A Domain Decomposition Method for Multivariate Spline Solutions of the Poisson Equation

Abstract

We explain a new kind of domain decomposition method (DDM) to discretize the 2D/3D Poisson equation based on continuous multivariate splines of arbitrary degrees into a linear system which can be solved based on a mix of direct and iterative methods. A Gauss-Seidel DDM algorithm and a Gauss-Jacobi DDM algorithm are proposed and the convergence of these two algorithms will be established. The classic SOR and two-grid techniques are used to accelerate the iterations. This new DDM divides a domain of interest into two or more subdomains without overlapping. Our domain decomposition method allows a parallel computation to further reduce the computational time. These algorithms are successfully implemented and tested. Numerical results will be presented to demonstrate the effectiveness and efficiency of the DDM approach.

Kirsten Morris

An Introduction to Topological Data Analysis (TDA)

Abstract

Topological Data Analysis (TDA) is a fairly recent approach to handling data sets using techniques from topology and geometry. In this talk I will provide an introduction to typical methods used in TDA and some contexts in which TDA has been applied.

Alex Newman

TBA

Abstract

Bill Olsen

TBA

Abstract

Eric Perkerson

Learning Low-Dimensional Manifolds

Abstract

Given a set of points $\{y^i\}_{i=1}^n$ all taken from some k-dimensional manifold M embedded in $\mathbb{R}^N$, our goal is to "learn" the structure of the manifold. For this talk, this will mean learning a classification function $\mathcal{C}\colon \mathbb{R}^N \to \{0,1\}$ that acts like an indicator function for the manifold M, returning $\mathcal{C}(x) = 1$ if $x \in M$ and returning $\mathcal{C}(x) = 0$ if $x \notin M$. We will discuss various approaches to this problem, including autoencoders that learn a parametrization of M, approximations to the expected value of $\mathcal{C}(x)$, and a new approach that learns the distance from a purposely corrupted data point $x + \eta u$ to the manifold M.

Freddy Saia

A Conjecture of Kac-Wakimoto and Modular Forms

Abstract

The classical number-theoretic problem of representing a positive integer m by sums of k triangular numbers has seen work from the likes of Gauss, Legendre and others. In 1994, Kac and Wakimoto were led to a breakthrough conjecture for the number of such representations for k of the form $4s^2$, through results stemming from the theory of affine superalgebras. Milne's proof of this conjecture using elliptic functions appeared two years later, but in this talk we will discuss a simpler proof by Zagier using the theory of modular forms.

Nolan Schock

Approaches to Enumerative Problems

Abstract

A typical problem in enumerative geometry asks, how many geometric objects satisfy a certain set of conditions? It is often easy to obtain an incorrect answer to such a question, and difficult to obtain a correct one. I will explain one well-known example of this phenomenon, the five-conic problem, and discuss several possible approaches to obtaining the correct answer to a given enumerative problem.

Zhaiming Shen

The Effectiveness of Quasi-Orthogonal Matching Pursuit Algorithm

Abstract

Classical orthogonal matching pursuit (OMP) algorithm has its merit in the effectiveness and efficiency of recovering the sparse solution of an under-determined linear system. I will introduce a new Quasi-orthogonal matching pursuit (QOMP) algorithm, which at some cost of computational complexity, will have a much better performance than OMP.

Arvind Suresh

Multiplication by 2 on elliptic curves

Abstract

For an elliptic curve E over a field K (of char different from 2,3), there is a classical theorem characterizing the group $2E(K)$: if E is given by $y^2 = (x-a)(x-b)(x-c)$, then $(x0, y0)$ is in $2E(K)$ if and only if $(x0-a)$, $(x0-b)$, and $(x0-c)$ are all squares in K. We present a proof of this theorem, which can be tweaked to produce a very similar characterization of $2E(K)$ for E given by an equation of the form $y^2 = f(x)$, with f monic and quartic.

Ben Tighe

A very brief introduction to deformation theory

Abstract

Deformation theory is the study of how schemes move in flat families (or, analogously, how complex manifolds move in holomorphic families). According to Kawamata's T1-lifting property, the most useful way to study deformations of schemes is to study *infinitesimal deformations*. In this talk, we will describe (infinitesimal) deformation theory from a functorial viewpoint and show how formalizing deformation functors — by Schlessinger's criterion — can classify the whole theory.

Jack Wagner

Local Systems and the Fundamental Group

Abstract

This talk will give a brief introduction to sheaves and local systems followed by an application to the fundamental group.

Haiyang Wang

Lenstra elliptic-curve factorization

Abstract

The fundamental theorem of arithmetic says that every integer can be written as a product of primes in a unique way. Currently, there are several efficient factorization algorithms. In this talk, I will discuss one of these methods using elliptic curves developed by Hendrik Lenstra in the mid-1980s.

Peter Woolfitt

So You Think You Can Multiply?

Abstract

The elementary school algorithm for multiplying two n digit numbers takes $O\left(n^2\right)$ operations. It turns out comboing the discrete Fourier transform with a divide and conquer approach can give a massive speed-up which reduces the computation to $O(n \log(n))$ operations. In this talk, we'll see how that works.

Xian Wu

ADE Dynkin Diagrams and Related Topics

Abstract

The ADE type Dynkin diagrams (graphs) are involved in Lie groups/algebras, algebraic geometry, operator theory, quiver representations, etc. I will give an overview of how these graphs give the classification of certain math objects.