Mock AMS Conference 2017

July 26 – July 27

Abstracts

Kübra Benli
Counting Pure Fields

Abstract

For an odd prime p and a number field K of degree p, K is said to be a pure field of degree p if for a p-free positive integer $n > 1$ we can write $K = Q(\sqrt[p]{n})$. Using an associated Dirichlet series, we present a method to count the number of pure fields of degree p whose discriminant does not exceed X.

Jordan Clark
Platonic Solids and Euler Characteristic

Abstract

We will explore applying Euler's formula to the sphere. Using this we will show that every polyhedron has either triangular faces or trivalent vertexes. Then we will show that with no triangles or 4-gons that we must have at least 12 pentagons.

Riley Ellis
Categories and the Cayley Embedding Theorem

Abstract

Categories are an incredibly powerful tool and language that rarely receive more than a passing mention in any course ("You've seen them in other courses, right?So moving on..."); Cayley's Embedding Theorem is a revered—yet useless(-looking)—theorem covered in every course on Group Theory, and even manages to pop-up in some form or another on quals. The aim of this talk is to give a basic overview of Category Theory, and explain how a number of common results—Cayley's Theorem incuded—are special cases of the fundamental tool known as Yoneda's Lemma.

David Galban
An Introduction to the Axiom of Determinacy

Abstract

Let $A \subset N^N$, the set of infinite sequences on the natural numbers. A game G_A involves two players consecutively selecting natural numbers a_n and b_n to form a se-quence

$$s = \langle a_0, b_0, a_1, b_1, \ldots \rangle$$

with Player I winning if $s \in A$ and Player II winning otherwise. If either Player I or II has a winning strategy for G_A, we say that G_A is determined, with the axiom of determinacy (AD) stating that for all such subsets A of N^N, G_A is determined.

Though AD is inconsistent with the axiom of choice, it is nevertheless consistent with the remainder of the Zermelo-Fraenkel axioms for set theory and has many sur-prising consequences for analysis. Here we discuss some of its implications, including countable choice on R and the Lebesgue measurability of all sets of reals.

Saurabh Gosavi
Essential dimension of symmetric groups

Abstract

Essential dimension of a collection of algebraic structures(finite dimensional alge-bra, vector space with a bilinear form etc.), informally speaking is the number of "independent parameters" required to describe the most general algebraic structure. For example: if K/k is a quadratic extension, then we know that (if char(k) $\neq 2$)

$K = k(\sqrt{a})$ for some non-square a in k and $\{1, \sqrt{a}\}$ is a basis for K/k. Thus, we need one "parameter", namely a to describe a general quadratic extension. What if K/k is a degree 3 separable field extension, or more generally a degree n extension? How many such independent parameters do we need? The problem is open in general. In this talk, we will answer this question for some small values of n.

Ernest Guico
Galois Theory: Forwards and Backwards

Abstract

The automorphism group of a field extension allows us to measure, in some sense, the extensions symmetry. In this regard, the field extensions with maximal symmetry are the so-called Galois extensions, and it is natural to ask which finite groups arise as the Galois groups of such extensions. One soon realizes that this is relatively easy if we allow the base field to vary with the group (in which case we find that every finite group is allowed). However, the problem (which is often referred to as the Inverse Galois Problem) remains open when the base field is fixed to be the rational numbers. In this talk, we will explore the Inverse Galois Problem over function fields of curves which will enable us to use tools beyond the usual ones from algebra.

Jason Joseph
Quandle Colorings and Knotted Surfaces

Abstract

Much of knot theory involves finding algebraic invariants of knots. Many of these invariants are extracted from the fundamental group of the complement of a knot. Tucked away inside this group lies a hidden structure which encodes the geometry of the knot much more naturally. These algebraic structures are called quandles, and they can be used to study any codimension two embedding. We will see how their axioms generalize the Reidemeister moves, and some of the slew of invariants that can be created with them. In particular, we will demonstrate a new way to calculate the tricolorings of a knotted surface in 4-space utilizing bridge trisections.

Andrew Maurer
Representation Theory via Geometry

Abstract

Given a finite group and a field F, we can define its cohomology groups $H^n(G; F)$, which turns out to be isomorphic to a certain set of somewhat-short exact sequences. This is used to define a product structure on $H^n(G; F)$. A classical theorem states this ring is finitely generated as an F-algebra – this allows us to make use of the tools of algebraic geometry. We will discuss the following question: What representation theoretic information does the variety associated to $H^\bullet(G; F)$ carry?

Clayton Mersmann
Spline Solutions to the Maxwell Equations

Abstract

The Maxwell equations are a system of first-order PDEs that describe all classical electromagnetic phenomena. We review common formulations of these equations for numerical study, and present a simple potential formulation for our spline analysis. In contrast with common finite element schemes, our spline solutions have an arbitrary degree of global smoothness; thus, we can obtain an accurate approximation of the electric and magnetic field quantities in question. Additionally, we use modified spline smoothness conditions to explicitly enforce discontinuities at material interfaces.

We conclude with some preliminary numerical results, including a computation of the electric field arising from a shielded microstrip.

William Olsen
Naturality in Heegaard Floer Homology

Abstract

The Heegaard Floer homology package, introduced by Ozsvath and Szabo in the early 2000's, provides powerful invariants. To a closed, oriented 3-manifold with a chosen $Spin^c$ structure they associate a (graded) abelian group. Initially, this group is only well-defined up to isomorphism, but recent work of Juhasz and D. Thurston shows how to naturally associate such a group. We address that issue in this presentation.

Hans Parshall
Small sumsets

Abstract

If A is a finite subset of an abelian group G, its sumset is defined to be $A + A = \{a + b : a, b \in A\}$. It is not hard to see that if $|A + A| = |A|$, then A is a coset. We will discuss some ways in which A behaves like an "approximate coset" when $|A + A|$ is "small".

Eric Perkerson
Reevaluative Subspace Clustering

Abstract

The reevaluative subspace clustering algorithm is an algorithm to solve the sub-space clustering problem, i.e. the problem of taking data vectors corrupted by noise, errors, and erasures from a union of subspaces and clustering the data by subspace membership. The algorithm consists of an initial subspace clustering algorithm, fol-lowed by a decomposition of each set of clustered vectors into a highly reliable set and its complement. Once sets of highly reliable sets are available for each cluster, the subspaces themselves are estimated from the data. With estimates of the subspaces, each data point's membership is reevaluated.

Makoto Suwama
SIC-POVM and abelian extensions of real quadratic fields

Abstract

A SIC-POVM is a set of d^2 equiangular lines in C^d through the origin which arises in quantum mechanics. In this talk, we will discuss its unexpected connection to real quadratic fields.

Luca Schaffler
Hyperbolic spaces and reflections

Abstract

The aim of this talk is to understand the mathematics behind tilings of hyperbolic spaces, which inspired some famous artworks of Escher. After a brief introduction to lattice theory, we define the hyperbolic space, and we explain how to subdivide it using reflection groups. Finally, we discuss how all this is used in algebraic geometry to construct compactifications of certain moduli spaces.

Bret Stevenson
Fixed Points on the Sphere and Hofer Distance

Abstract

A rotation of the sphere is an example of a Hamiltonian diffeomorphism. The collection of all such diffeomorphisms is denoted by $\mathrm{Ham}(S^2, \omega)$, where ω is an area form on S^2, and like rotations, every $\varphi \in \mathrm{Ham}(S^2, \omega)$ has at least two fixed points (Simon, '74; Nikishin, '74). Moreover, the group $\mathrm{Ham}(S^2, \omega)$ comes with a metric d_H called Hofer's metric. The question becomes: How far away is any $\varphi \in \mathrm{Ham}(S^2, \omega)$ from some $\psi \in \mathrm{Ham}(S^2, \omega)$ with the minimal number of fixed points? This talk will present a partial answer to this question.

James M. Taylor
Nimbers and the Sprague-Grundy Theorem

Abstract

Nim is an impartial game in which two players take turns removing objects from a finite number of heaps until all heaps are empty. A players turn consists of choosing one heap and then removing at least one object from it. The player who removes the last object wins the game.

A single heap game of Nim is as simple as it gets; such games can be described by a unique ordinal number *n, called a nimber, which gives the size of the heap. In this talk, we will prove the Sprague-Grundy theorem, which states that every impartial game under the normal play convention (last move wins) is equivalent to a unique nimber, and hence to a Nim game with only one heap.

Abraham Varghese
The Netflix Problem

Abstract

This talk will introduce you to the Netflix Problem, the mathematics behind it and a brief survey of the existing algorithms to solve it.

Zerotti Woods
A Brief Introduction To Information Theory

Abstract

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. (Claude Shannon, 1948) In this talk we will discuss ways to achieve nearly perfect communication over an imperfect, noisy communication channel. We will also discuss the limitations and the possibilities of communication over noisy channels. I will attempt to make this talk very friendly by giving a few simple examples.

Peter Woolfitt
A Fourier Analytic Approach to the Cap Set Problem

Abstract

Recent exciting work has blown away longstanding upper bounds on the size of cap sets. I shall not be talking about that work. Instead, I'll be discussing an older method of Meshulam using Fourier analysis to get nontrivial bounds on the size of cap sets. The goal of this talk is to give you a taste of the power of the Fourier analytic approach which is still (at heart) used to get the current best bounds for similar problems in other settings.

Xian Wu
Topological Mirror Symmetry

Abstract

I am going to talk about Batyrev's construction of mirror symmetry via hypersur-faces in some special toric varieties.

Ziqing Xiang
Dimensions of Specht modules

Abstract

It is well-known that the dimension of the Specht module of a partition equals to the number of standard Young tableaux whose shape is the given partition, and the latter one can be computed using hook length formula. In this talk, I will show the integer factorization of this number and discuss its application in representation theory.

Yidong Xu
The Minimum of a Convex Function

Abstract

The great importance of extremum problems in applied mathematics leads one to the general study of the minimum or maximum of a function over a set C. When a sufficient amount of convexity is assumed, the study is greatly simplified, and many significant theorems can be established. These results not only have theoretical values but also present practical meanings.

Nathaniel Zbacnik
Counting Weighted Compactifications of $\overline{M}_{0,n}$

Abstract

In 2002, Brendan Hassett introduced the notion of moduli spaces of weighted pointed stable curves, which gives numerous ways to compactify the moduli space of pointed curves. This talk will explore some of the ways one can try to count these compactifications, which leads to some fun combinatorial games.

The Wake/Davidson Experience in Number Theory Research, Group I:

C. Donnay, H. Ellers, K. O'Connor, E. Wood

Intersecting Finite Sets of Integral Positive-Definite Binary Quadratic Forms

Abstract

Every quadratic form represents 0; therefore, if you take any number of quadratic forms and ask which integers are simultaneously represented by all members of the collection, you're guaranteed a nonempty set. But when is that set more than just the "trivial" {0}? In this talk, we address this question in the case of integral, positive-definite, reduced, binary quadratic forms. For forms of the same discriminant, we can use the structure of the underlying class group. If, however, the forms have different discriminants, we must apply class field theory.

The Wake/Davidson Experience in Number Theory Research, Group II:

S. Binegar, R. Dominick, M. Kenney, A. Walsh

An Elliptic Curve Analogue to the Fermat Numbers

Abstract

The familiar Fermat numbers, which take the form $2^{2^n} + 1$, have several notable properties, including order universality, coprimality and definition by a recurrence rela-tion. Our goal is to define an analogous sequence generated by points on elliptic curves, looking in particular at the denominators of the x-coordinates. In order to show that our sequence is indeed analogous, we prove analogues of results of the standard Fermat numbers. We have also discovered exciting patterns within the elliptic Fermat sequence of specific points and curves.