

## Math 8190: Algebraic Groups, Fall 2016

Description: We will study affine (or linear) algebraic groups over a field, usually algebraically closed, with an emphasis on key aspects of the structure of such groups and their Lie algebras. This course will differ substantially from its most recent offering (Fall of 2014 with Dr. Nakano). Topics to be covered:

- Basics: Jordan decomposition, Lie algebras, Hopf algebras,  $G$ -varieties, quotients by subgroups, homogeneous spaces, affine group schemes.
- General Structure Theory: connected solvable groups, tori, connected unipotent groups.
- Reductive and Semisimple Groups: isomorphism type of connected group determined by root datum, the Bruhat decomposition, the flag variety  $G/B$ , the Jacobson-Morozov Theorem and variants of it in positive characteristic, isomorphisms between the nilpotent variety of Lie algebra and unipotent variety of algebraic group, Bala-Carter-Pommerening classification of nilpotent and unipotent orbits.
- Geometric Invariant Theory: geometric reductivity, categorical and geometric quotients, actions of a reductive group on an affine space.

**Prerequisites** - We will aim to keep this course accessible to as many students as possible. A reasonable knowledge of graduate algebra (groups, rings, fields, modules) will be assumed. Familiarity with algebraic geometry is helpful, but not required.